Physical Emergence,
Diachronic and Synchronic

Introduction

Writing in the heyday of what has come to be known as 'British Emergentism', Samuel Alexander suggested that

> the emergence of a new quality from any level of existence means that at that level there comes into being a certain constellation or collocation of the motions belonging to that level, and this collocation possesses a new quality distinctive of the higher-complex. [...] To adopt the ancient distinction of form and matter, the kind of existent from which the new quality emerges is the 'matter' which assumes a certain complexity of configuration and to this pattern or universal corresponds the new emergent quality. (1920, 45, 47)

This can be taken[1] as characterizing a notion of an emergent property as a property that is prossessed by a whole, or configuration, of constituent parts such that none of the parts have that property on their own and that the property is characteristic of this (kind of) configuration. In other words, emergent properties, on this view, appear to coincide with what more recently has been called structural properties: P is a structural (higher-level) property iff P is the property of having proper parts a_1, a_2, ..., a_n , such that there are lower-level properties $P_1(a_1)$, $P_2(a_2)$, ..., $P_n(a_n)$ and the parts stand in relation $R(a_1, a_2, ..., a_n)$. (Kim 1998, 84) Such a P is 'new' in the sense that none of the properties of the parts, P_i ($i = 1, ..., n$), is identical with it; P is characteristic of the configuration of the parts a_i because it depends on the specific relation R in which the parts stand. Furthermore, P can obviously be equipped with causal powers which are different from the causal powers of any of the P_i . Putting three 3kg pieces of metal together results in a configuration with the 'new' property of weighing 9kg with different causal powers than the individual pieces had.

Emergentists, however, do not tolerate a case like this as an example of an emergent property. Usually such a property would be labelled a 'resultant' property. How do we distinguish emergent from resultant properties? One way is to spell out the 'novelty' of emergent properties as their being unpredictable, unexplainable, or irreducible (in some sense): a description of the lower level properties and their configuration will not be sufficient to allow us to predict or derive the new higher-level properties, or explain how they resulted from their bases, or reduce them to these bases. Note that this condition, which presumably is violated in the 9kg-case, could still be compatible with the definition of a structural property as given. Lets call this "weak emergence" (Bedau 1997) or the "innocent" view of emergence (Chalmers 1996, 378, n.41).

Emergentists have usually set the standards somewhat higher, though. A structural property, even if we grant it to be unpredictable or irreducible (in some sense), will still be nothing 'over and above' (to use a characteristic emergentist phrase) the property of a set of parts configurated in a certain way; its causal powers will not be different from the causal powers the configuration has in virtue of the parts being configured in this way. Thus, one could argue that emergent properties should supervene on the structural properties of wholes such that the causal powers of the former are different from the causal powers of the latter. This is 'strong emergence'. The trouble with strengthening the notion this far is, as Kim has argued, that the strongly emergent properties get involved in an intolerable causal competition with their structural base properties (e.g., Kim 1999, 31-

33).

The task of this paper is to explicate a notion of weak emergence for physical properties. I would like to show, in a first step, that we can make good sense of a distinction within resultant or structural properties: we can differentiate those properties which are 'novel' from those which are 'merely resultant'. I suggest a way of spelling out what it is that makes the property of weighing 9kg a merely resultant property of the lower-level properties of three pieces of metal, and what makes the dynamic behaviour of a heart novel with respect to the behaviour of an undamped harmonic oscillator and why the heart-like behaviour is not reducible to the ideal oscillator behaviour. Thus, even if we accept the restriction to structural properties, I argue, we can reconstruct a sense of diachronic or evolutionary emergence. In a second step I propose a reconstruction of a notion of synchronic emergence along similar lines which satisfies many of the traditional desiderata without introducing strongly emergent properties.

Everything here is physical; I am not trying to analyze the emergence of non-physical properties out of a physical basis. In fact, a classic strategy of making the notion of emergent properties less mysterious and more palatable — even to logical positivists — , was to show that such properties are actually quite common in the domain of the physical sciences (e.g., Feigl 1958, 414f.). Recently this strategy has been applied again by a number of authors (Humphreys 1997, Newman 1996, Wimsatt 1996). I shall look at some of these views on emergence and argue that the various requirements set up for emergent properties — in particular, novelty and irreducibility — can be satisfied in fairly simple and common systems in physical science if we avail ourselves of some of the tools of dynamical systems theory. My claim is that the requirements for emergent properties have natural and fairly precise counterparts in this part of physics and that a unified account of these requirements becomes possible.

1. Levels of physical properties, types of emergence

The criteria for emergent properties I take from the literature as more or less canonical are: (i) novelty of the properties in a system, (ii) that the properties result from 'essential interactions' of constituent parts of a system, and (iii) that the laws governing the properties be irreducible to laws about lower-level properties (cf., for instance, Stephan 1992, Humphreys 1996, Kim 1999). I'll concentrate on (i) and (iii); condition (ii) is briefly discussed in Appendix I.

Before the requirements can be implemented, we need to specify the way in which a hierarchy of levels of properties is to be identified such that we can extend talk of, e.g., biological or psychological properties — as higher-level properties — being emergent with respect to physical — lower-level — properties, to properties within the physical realm alone. Most frequently levels of physical properties have been distinguished according to mereological relations; among the possibilities in this domain, the most common one is to separate a macrophysical (higher) level from a microphysical (lower) level.

The mereological level distinction is relevant for the case of synchronic emergence of properties; I will later adopt a version of it — a separation of system and sub-systems — in Section 3. For diachronic emergence, however, the relevant distinction is between the properties of a system at one time and those at a later time. The system at a time I describe by the variables used in dynamical systems theory: some set of 'generalized coordinates' — position and momentum — , which take on a sequence of values as time passes, a sequence represented as a trajectory in the system's phase space portrait. In addition to the coordinates we need one or more 'control parameters' which specify features of the system, like friction, which are not assumed to be determined by the system's internal dynamics. The phase space trajectories express the behaviour of the system. For instance, a behavioural property of a damped oscillator is that its motion gradually winds down and ends in a rest state while an undamped oscillator has the property of continuing its motion indefinitely. We could express things perhaps more precisely as follows: The property of having its motion wind down (call it 'N', a second-order property) can be 'functionalized' as the property of a system showing such and such long-term behaviour, given such and such input; we then find a structural (first-order) property which 'realizes' N, namely, the property of being a system with its variables and parameters arranged in the way characteristic of a damped harmonic oscillator.[2]

We can say that the behaviour of the system at a time is emergent with respect to the system at an earlier time if some parameter in the base has changed its value slightly during the time interval and the later behaviour is 'novel' compared to the behaviour of the old system and irreducible to it. This is evolutionary or diachronic emergence of properties. The general strategy in developing this notion within the framework of dynamical systems theory is to compare two systems which are connected through a small change in the base properties such that the 'old' system is an unperturbed version of the 'new', perturbed system, and find out whether this quantitatively small perturbation of the base leads to qualitatively new behavioural properties.

In Section 3, in order to reconstruct a notion of synchronic emergence, I will apply the same strategy to a system at one time, now dividing the levels mereologicly into properties of the whole system and properties of sub-systems.

2. Diachronic emergence in a dynamical system

2.1. Novelty

Novelty is probably the most common as well as the most elusive of the conditions for emergent properties. That a property should be novel in order to count as emergent is usually motivated by pointing out that, e.g., acquiring a new value for a property like 'weighing 10kg' instead of 'weighing 9kg', is not enough for this 'new' property to qualify as emergent. The property of having mass, the determinable, has been there all along; only its determinates changed. 'Novelty of property P in object x' is thus contrasted with 'P is a determinate of a determinable p such that the constituent parts of x have determinates of p different from P'. I will illustrate with the following example which will serve throughout as a paradigmatic case of the application of dynamical systems theory.

Consider as our system a damped nonlinear oscillator, realized, for instance, by a triode circuit involving a nonlinear resistor with a cubic voltage-current characteristic. Let the amplitude of the oscillations, $x(t)$, be given by van der Pol's equation

$$x'' - \eta(1 - x^2)x' + x = 0 \quad (1)$$

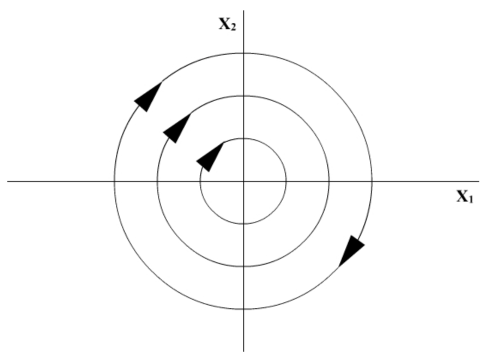
with $x' = dx/dt$ and η is a parameter measuring the strength of the damping term $-(1-x^2)x'$. For large amplitudes x this factor becomes positive, thus keeping the oscillations bounded; for small amplitudes the term is negative ('negative friction') and thereby excites the oscillatory movement. Systems described by equations of the van der Pol type are therefore sometimes called ‘self-exciting’, even though ‘negative friction’, of course, means the injection of energy from outside the system.[3] Equation (1) is equivalent to a system of first-order differential equations:

$$x_{1'} = x_2 \tag{2a}$$

$$x_{2'} = \eta(1 - x_1^2)\, x_2 - x_1 \tag{2b}$$

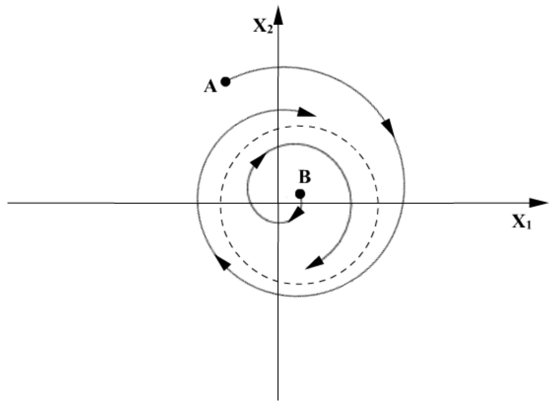
which are convenient for constructing phase space portraits of the system with x_1 and x_2 as phase space coordinates.

Suppose the damping could be reduced to zero ($\eta = 0$). We would then have an undamped harmonic oscillator with the familiar phase portrait of a system of concentric ellipses (fig.1).



(Fig.1)

From whatever initial conditions (x_1 , x_2) you start the system, it will oscillate such that it periodically passes through these same conditions. In the language of dynamical systems theory: the system has as its equilibrium point a ‘center’ at the origin ($x_1 = 0$, $x_2 = 0$). If we gradually turn on the damping, the system will still show oscillations (fig.2):



(Fig.2)

Now, however, the oscillations have a 'limit cycle', that is, from wherever you start the oscillator (arbitrary initial conditions α and β, excluding the rest state $x_1 = x_2 = 0$)), the system will tend towards a unique periodic behaviour, the cycle shown in the figure as a dashed closed curve. Such limit cycle behaviour is characteristic of many systems in nature, and one of the early applications of the van der Pol equation was in modelling the limit cycle behaviour of the heart.

Clearly, this system has the same basic determinables in the undamped and in the damped regime, in particular, position (x_1) and momentum (x_2). Only the succession of values which these determinables take on changes. The fact that the damping increases from zero to a finite value must not be taken as indication that a new determinable (viz., damping) has been introduced into the system. Nothing hangs on the particular value of zero for η. The kind of qualitative change in behaviour we are interested in occurs in general at a 'critical value' of a parameter, whether this is zero or some other value. Although no new determinables have been introduced into the set of 'base' properties of the system compared to the base with $\eta = 0$, the limit cycle behaviour of the oscillator in the damped regime is a novel property, a feature which distinguishes the system with damping qualitatively from the system without damping.

This feature of novelty of a property is a traditional ingredient in the notion of emergent properties and is captured in our framework by the fact that the phase space portraits in figs.1 and 2 are 'topologically inequivalent': no smooth deformation will transform the set of trajectories of Figure 11 into those of Figure 22, even though, for small enough damping, the trajectories of the damped system can be quantitatively very close to those of the undamped system. In other words, the oscillator is 'structurally unstable' around the critical value of the control parameter η; small variations of the parameter (perturbations of the system) will lead to qualitatively different behaviour. At the critical value of η the system undergoes a 'bifurcation'. Conversely, if the phase space portrait stays qualitatively the same under perturbations of the dynamics, i.e., small variations in the value of the control parameter, the system is 'structurally stable'.[4]

We then have, in the language of dynamical systems theory, the following picture for what happens when a 'novel' property appears in the diachronic case (Rueger, ms.):

A system with base properties A, characterized by a value of a control parameter *p*, under slight variation of the parameter around its bifurcation value, turns into a system with base properties A*, which shows qualitatively different behaviour B* than the original system with behaviourial characteristic B. The original system with {A, B} I call the reference system for the system with {A*, B*}. A property **b** ∈ B* in a system {A*, B*} is 'novel' if

(1) reference systems {A, B} with bases A different from A* only in the value of the control parameter p, do not have **b**; and

(2) the behaviour of {A*, B*} in which **b** manifests itself (in the phase space portrait) is qualitatively different from, or topologically inequivalent to, the behaviour of {A, B}.

Thus, the novel property of the van der Pol oscillator with $\eta > 0$ considered above would be 'having a limit cycle', a feature missing from the oscillator with $\eta = 0$. Contrast the evolution of the system from $\eta = 0$ to a slightly different, positive value for η with the evolution of the system in a different parameter regime, say from $\eta = 0.5$ to a slightly higher value: There is a clear sense in which the first parameter change (around the bifurcation value) is accompanied by novel behaviour even though, of course, the behaviour of the system does change through the second parameter modification as well (the period of the limit cycle, for instance, will change slightly). Although the behaviour in each case can be regarded as a structural property of the system (or as supervening on such a property), it is obvious that the second change leads to a merely resultant property while the first transition generates a novel property.

Note that the notion of novelty is defined as a relation — novelty with respect to a reference system — and that it is a relation between systems at different times ({A, B} and {A*, B*}) rather than a relation between properties of a system at one time. It is also important to note that even though the relation of being topologically inequivalent, or qualitatively different, is symmetric, the symmetry is broken by choosing one system as a reference system.

A related way of phrasing requirements for novelty — or, in this case, emergence — is Wimsatt's condition of 'violations of aggregativity' (Wimsatt 1976, 1996; Bechtel/Richardson 1992). This notion is supposed to capture the intuition that "the whole is more than the sum of its parts" without making any claims, according to Wimsatt, about the properties of the whole not being reducible to properties of, and laws about, the constituent parts. An emergent property, for Wimsatt, is a property of a system which is dependent on the mode of organization of the system's parts, how they are aggregated into the whole. Properties which are invariant against (small) changes in the modes of aggregation are non-emergent. In order for a system property P to count as emergent, P has to violate some or all of the following conditions of aggregativity:

(i) P is invariant under rearrangements of parts of the system or replacements with "relevantly equivalent" parts;

(ii) P is "qualitatively similar" to the property exhibited by the system when parts are added or subtracted;

(iii) P is invariant under decomposition and reaggregtion of parts;

(iv) there are no "cooperative or inhibitory interactions" involving P among the parts of the system.

It should be clear how the notion of structural stability is able to capture Wimsatt's robustness or invariance against modifications in the mode of aggregation of the system's parts. If we choose, for instance, according to condition (i), in the van der Pol oscillator case as control parameter the damping, we can consider small variations of the parameter as a result of parts of the system (resistor, capacitance, etc.) being replaced by similar parts. If the system's phase space portrait stays qualitatively the same under such perturbations, the system's property under consideration satisfies the aggregativity condition (i). Failure of structural stability means violation of aggregativity and hence emergence (in Wimsatt's sense) of some novel property in the system. What our framework adds to Wimsatt's — besides greater generality — is a criterion for qualitative similarity, that is, a way of telling when a system property, represented in the system's phase space portrait, is invariant or not.[5]

2.2. Nonreducibility

The appearance of novel properties in a dynamic system can be shown to be connected with a sense of nonreducibility of the new property to the properties of the reference system. Non-reducibility, besides novelty, has been another traditional ingredient in notions of emergence.

The question of reducibility will be phrased in terms of descriptions of, or theories about, the properties of the systems and a notion of reduction as a relation between such descriptions or theories has to be chosen. We take a fairly liberal approach, not as demanding as deducibility of one theory from another (as in Nagel-type reductions). We use a notion of reduction that, as Nickles put it, is common to "physicists and mathematicians, in contrast to most philosophers" (Nickles 1973, 182) who usually have focussed on deriving the less general from the more general theory, thereby 'reducing' (in the philosopher's sense) the former to the latter. The physicist's sense of reduction, labelled 'reduction$_2$' by Nickles, is the inverse: the more general theory is said to reduce to the less general theory in the limit of a certain parameter. Thus, a theory or theory part which we assume to be formulated as a function of variable(s) x and parameter(s) p, Θ'(x; p), reduces to another theory or theory part Θ(x; p=0) if

$$lim_{p\to 0}\, \Theta'(x;\, p) = \Theta(x;\, p=0),$$

under an appropriate choice of parameter p for the limit and <u>for all values of x</u> (for which Θ' is defined). More precisely, the solutions to the equations of motions of Θ', in the limit of $p \to 0$ (or some other suitable value), should <u>converge uniformly</u> (not just pointwise) to the solutions of the equations of Θ, where we set p = 0. Special Relativity Theory (Θ'), for example, is reducible in this sense to Classical Mechanics (Θ) because Θ' goes smoothly over into Θ in the limit of 'small velocities', $(v/c)^2 \to 0$ (cf. in particular Batterman 1995; 1997; Berry 1994; Primas 1983, ch.6). Thus, the expression for relativistic momentum reduces uniformly to the formula for classical

$$\lim_{\frac{v^2}{c^2}\to 0} \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}} = m_0 v$$

momentum:

where m_0 is the rest mass. Note that if Θ' is not reducible to Θ in this way, then there will also be a failure of (Nagel-) reduction in the sense of a deduction of Θ from Θ'.

It is sometimes claimed that the limit-notion of reduction is applicable only to theories within the same level — 'intralevel reductions' — while the model fails to account for 'interlevel reductions', e.g., the reduction of Thermodynamics (macro-level) to Statistical Mechanics (micro-level) (cf. Wimsatt 1976, 216f.). I think this is mistaken. Although, given the micro-macro-level hierarchy, the example of Special Relativity Theory and Classical Mechanics would count as a case of intralevel reduction, the interlevel relation between Thermodynamics and Statistical Mechanics can just as well be fruitfully studied with the limit-notion. In fact, the usual treatment of the relation is in terms of the 'thermodynamic limit' of Statistical Mechanics, a singular limit, which allows at least a partial understanding of how thermodynamic properties 'emerge' from Statistical Mechanics in the limit.[6]

The cases I discuss, a damped nonlinear oscillator and the undamped harmonic oscillator, would be classified as intralevel rather than interlevel. The 'direction' of emergence is therefore not given by the micro-macro hierarchy of levels but rather has to be determined by other means: in the case of diachronic emergence, the direction is given by the direction of time; for synchronic emergence (Section 3, below) the direction will be determined by the decomposition of a given system into subsystems. In either case the more general theory describes novel properties compared to the less general theory; starting with the undamped harmonic oscillator (either as the temporally earlier system or as the subsystem of a given system) we see the emergence of a novel property upon varying the damping parameter slightly.

The question of reducibility in the case of diachronic emergence takes on the form: Can a description of the system {A*, B*} (corresponding to Θ') be reduced to a description of the reference system {A, B} (corresponding to Θ), i.e., does Θ' smoothly go over into Θ in the appropriate limit of the control parameter?

2.2.1. The lightly damped van der Pol oscillator

The van der Pol oscillator, described by equation (1), serves again as our example of a dynamical system. For $\eta = 0$,van der Pol's equation reduces to the equation for a simple harmonic oscillator (see Figure 11) which is easy to solve. We could expect that for <u>small</u> damping we could describe the system by 'perturbing' the $\eta = 0$ solution, that is, adding 'small' corrections to the simple harmonic oscillator such that the general solution would be an expansion around the ($\eta = 0$)-solution, $x_{\eta = 0}(t)$, in powers of η:

(3) $$x(t) = x_{\eta=0}(t) + \eta\, \xi_1(t) + \eta^2\, \xi_2(t) + \ldots$$

If this could be done, i.e., if the van der Pol oscillator could be treated as a 'regular perturbation' of the simple harmonic oscillator, we would have

(4) $$\lim_{\eta \to 0} (solution\ for\ \eta \gg 1) = (solution\ for\ \eta = 0)$$

for all values of the independent variable, *t*, and our condition for reducibility, a uniform limit relation between the solutions, would be satisfied.

Inserting the expansion (3) into equation (1), however, leads to a power series expansion for *x(t)* which is asymptotic for fixed t as $\eta \to 0$ (i.e., the difference between the function-to-be-approximated and each partial sum of the series is smaller than the last term included in the sum, for small enough η), but which breaks down for large enough t. We arrive at an expansion for the amplitude of the form $x(t) = \cos \omega t + \text{const}\ \eta(t \cos \omega t) + \ldots$, where the first term is the solution for the undamped harmonic oscillator with frequency ω; the second term, however, is unbounded for $t \to \infty$ and thus destroys the asymptoticness of the expansion.[7] The appearance of such 'secular' terms is due to the nonlinearity of the system: the frequency of the oscillations of the nonlinear problem depends on the amplitude while there is no such dependence in the linear (undamped harmonic oscillator) problem. Expanding the solution around the linear solution imposes the amplitude-independent frequency ω on the system while in fact the nonlinear friction will slowly change the frequency.

Condition (4) thus cannot be satisfied in this case; the η-limit is not uniform — the case turns out to be a 'singular perturbation' problem. The theory of the lightly damped van der Pol oscillator cannot be reduced to the theory of the undamped case. The property of having a limit cycle therefore is novel in the sense defined above as well as irreducible to the properties possessed by the reference systems, the undamped oscillators.

We can see here an important correspondence between the notion of novelty as structural instability and the notion of irreducibility as singular limit relation: The radically different behaviour of the perturbed system compared to the behaviour of the unperturbed system, which gives rise to the singular limit relation between the two, is nothing but an expression of the fact that the unpertubed system is structurally unstable; any slight perturbation of this system will lead to systems which behave qualitatively differently. Thus, the regular perturbation approach breaks down in these cases; we have irreducibility of the perturbed to the unperturbed system. In short, every singular perturbation problem implies a structurally unstable (unperturbed) system and thus a transition to 'novel' behaviour (of the perturbed system) (cf., for instance, Guckenheimer/Holmes 1983, ch.4). In the following I will therefore no longer explicitly distinguish between irreducibility and novelty.

We have the following result for the lightly damped van der Pol oscillator: The behaviour of the system just past the bifurcation value of the damping parameter is 'novel' in the sense defined above; furthermore the description of this qualitatively new behaviour is not reducible, in the sense defined above, to the description of the behaviour of the pre-bifurcation system (the reference system). As a further illustration of the notion of emergence suggested here I mention the case of phase transitions, e.g., from gaseous to liquid in non-ideal gases or from the paramagnetic to the ferromagnetic phase in ferromagnets. Around the critical (or bifurcation) value of the control parameter (here the temperature), very small variations in the parameter value lead to qualitatively different behaviour and properties of the system (cf. Rueger, ms.).

3. Synchronic emergence

If we retain the basic constraint on our discussion — that the weakly emergent properties supervene on structural properties and can be functionalized — how can we modify the account of diachronic (weak) emergence into a reconstruction of synchronic (weak) emergence? I suggest distinguishing two levels of properties, a higher and a lower, such that the candidates for weakly emergent property status supervene on the higher level properties, which in turn are to be understood as structural properties, i.e., they are properties which a configuration of lower-level properties has in virtue of the fact that these properties (or their carriers) are configurated in a specific way. We can say, in functionalist fashion, that some property N supervening on the higher level properties is 'realized' by them. N is emergent if it is novel or irreducible, in the sense defined above, with respect to the properties at the lower level.

Frequently, at least in philosophical discussions, the property levels are distinguished as macro and micro levels. N, a property carried by the macro level, is emergent if the description of the system at the micro level does not have the description of the system at the macro level as a uniform limit (with some suitable parameter), that is, if the macro realization of N is not reducible to the micro level. In line with the illustrations used so far, however, I shall not employ the micro/macro level distinction but rather introduce a different kind of distinction which is also used widely in science. The almost exclusive focus of philosophical discussions on micro and macro levels I believe to be a prejudice because there are other, scientifically equally important strategies of separating levels in a system which deserve some attention. I shall come back briefly, however, to the traditional micro/macro case in Section 4

Suppose we have a theory which describes the behaviour of a system with the property of interest, N. We take this theory as describing a realization, at the higher level, of that property. Now suppose further that the theory is subjected to a perturbation expansion like in Equation (3): we decompose the full solutions x(t) of the equations of the theory, for small values of a perturbation parameter η, into a main contribution from an unperturbed sub-system and disturbances, of (hopefully) decreasing magnitude, of this main contribution from other factors present in the system:

$$x(t) = x_{\eta=0}(t) + \eta\, \xi_i(t) + \eta^2\, \xi_i(t) + \ldots$$

The $\xi_i(t)$, besides the sub-system $x_{\eta=0}(t)$, define the lower level of the property hierarchy. The system, described at the higher level by x(t), is decomposed into a configuration of sub-systems at the lower level which all contribute, more or less, to the higher level behaviour.

Taking a system's actual behaviour, the higher-level properties, and analyzing it as the result of the behaviour of an unperturbed system — the lower-level properties — together with the influence of 'small perturbations' onto this behaviour, is not an unusual or arbitrary choice of a way of decomposing a system into levels. In fact, there have been claims that this way of analyzing systems is the typical research strategy of modern science since Galileo (Cartwright 1989; cf. Rueger/Sharp 1998). However exaggarated such claims may be, what Galileo called the 'resolutive' approach in many contexts characterizes what we mean by 'understanding' how a system works. In the perturbation approach, we abstract away the 'core' from the 'total' behaviour of a system.[8] The knowledge of such core factors we then use in our attempts to account for the behaviour of other systems with different total behaviour: we try to find different arrangements of the factors which will reproduce the observed behaviour of these systems.

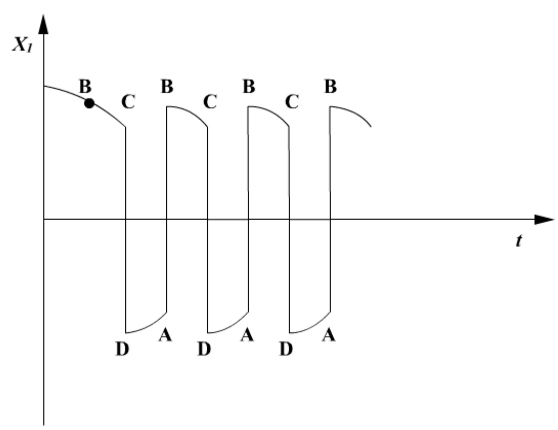
What significance does it have if the higher and lower level (descriptions) of a system — system and sub-systems — are related in the way in which theories are related in singular perturbation problems? Recall the discusson of diachronic emergence: there we analyzed the case where a description of a later state of the system does not reduce to a description of an earlier state. Now, for synchronic emergence, we have to study the case where a description of the system at the higher level does not reduce to a description at the lower level. That means, we have to distinguish those systems where $\lim_{\eta\to 0} x(t) = x_{\eta=0}(t)$, uniformly for all t, or, more explicityly, $\lim_{t\to 0} \lim_{\eta\to 0} x(t) = \lim_{\eta\to 0} \lim_{t\to 0} x(t)$, from those systems where these limit relations between the levels do not hold. Only in the former case (the regular case) can we assume that the behaviour of the full system x(t) will not stray far from the behaviour $x_{\eta=0}(t)$ for any t, that is, that $x_{\eta=0}(t)$, the main factor at the lower level, is a uniformly valid approximation of x(t) which can be improved upon by considering more terms in the perturbation expansion. When the uniform limit relations are violated (the singular case), x(t) can be very different from $x_{\eta=0}(t)$ in some range of t-values and the lower-level main factor cannot be said to uniformly approximate the full behaviour of the system. In the latter case, we have synchronically emergent properties at the higher level, in the former merely resultant properties.

For instance, if we take Special Relativity Theory to be the description of the system at the higher level, the realization of the property of having relativistic momentum would be described by $m_0 v\ (1 - v^2/c^2)^{-1/2}$. We can expand this expression, for 'small velocities', in a power series in the parameter $(v/c)^2 < 1$; the first term in this expansion is the description in terms of Classical Mechanics, $m_0 v$. On this way of distinguishing levels, the relativistic system has a 'classical sub-system' at the lower level. As we know, the descriptions satisfy the uniform limit condition and therefore $m_0 v$ is (in the specified parameter regime) a uniform approximation of the full relativistic system. (In systems in which $(v/c)^2$ is not 'small', classical momentum, of course, will no longer be a 'core realization' of relativistic momentum. The realization relation, in our reconstruction, thus turns out to be 'structure-specific', restricted to systems which are in the appropriate regime of the perturbation parameter.[9])

For synchronic emergence we consider the relation, at a given time, between the behaviour of the 'main part' of the system, if it were operating on its own, isolated from the rest of the system (the perturbations), and the behaviour of the total system, main part and perturbations together, i.e., the actual behaviour. If the latter is novel (in our sense) compared to the main-part behaviour and hence non-reducible (in our sense) to this behaviour, we have synchronic emergence. Examples of this type of emergence are, of course, provided again by the van der Pol oscillator discussed above. In the small damping regime, for instance, the main sub-system would be the undamped harmonic oscillator with $x_{\eta=0}$ (t), the behaviour of which is not a qualitatively good approximation to the limit-cycle behaviour of the full system. With respect to the perturbation decomposition of the system into levels, the limit-cycle behaviour would qualify as a a synchronically emergent property.

3.1. The heavily damped van der Pol oscillator

We now take the van der Pol oscillator in the regime of heavy damping (η >> 1). If we increase the damping strength, the oscillations of the system become jerky or 'almost discontinuous' (Figure 3):



(Fig. 3)

While we have, for small damping, oscillations that are quantitatively close to the behaviour of an undamped harmonic oscillator, large damping generates a very different type of behaviour, characterized by an alternation of 'slow' motion of the system along BC, DA, ... and 'fast' motion along CD, AB, ... It is as if the system builds up tension for a while and then suddenly relaxes ('relaxation oscillations'). For large η we cannot treat the damping as a small perturbation of the undamped behaviour. Equation (1) has to be transformed in order to apply a perturbative method of solving it (cf. Grasman 1987, 55ff. or Mishchenko/Rozov 1980, ch.1). Define a new parameter $\varepsilon = 1/\eta^2 << 1$, change the time variable t in equation (1) to $\tau = t/\eta$, and write

(5) $$\_\, x'' - (1 - x^2)\, x' + x = 0$$

where x' now denotes $dx/d\tau$. The equivalent first-order equations become

(6a) $$x_{1'} = x_2$$

(6b) $$\_\, x_{2'} = (1 - x_1^2)\, x_2 - x_1 = 0$$

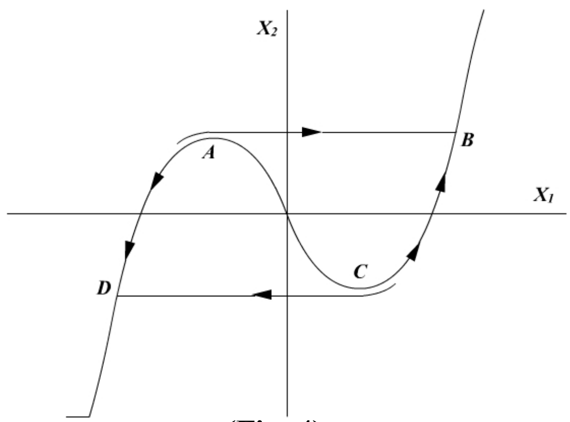
This will give us the description of the oscillator for $\varepsilon \rightarrow 0$, corresponding to $\eta \rightarrow \infty$. Call it theory Θ.

The case $\varepsilon = 0$, i.e.,

(7a) $$x_{1'} = x_2$$

(7b) $$(1 - x_1^2)\, x_2 - x_1 = 0$$

is the case of purely 'slow' motion, the description of the 'slow' sub-system. Call it theory Θ'. The phase portrait in the large damping regime looks like this (with letters corresponding to letters in Figure 3):



(Fig. 4)

Without actually solving the equations we can see that the condition for reducibility, $lim_{\eta \to 0}\, \Theta = \Theta'$, cannot be expected to be satisfied; the solutions of equations (7a,b), i.e., for ε = 0, will in general be different from the solutions of equations (6a,b) in the limit ε → 0 (but ≠ 0). This is so because the original equation (5) is a second-order differential equation which turns into a first-order equation for ε = 0; solutions of the latter can, for instance, satisfy only one initial condition, not the two conditions required for the second-order equation. In fact, the solutions for ε = 0 give us the 'slow' trajectories of the system in phase space along DA and CB which are not periodic. For ε > 0 (but << 1) the solutions are radically different since they describe periodic motion with a change between 'fast' and 'slow' sections. The slow sub-system, described by Θ', is not a regular or continuous limit of the system described by Θ; Θ' is a singular limit of Θ. We cannot analyze Θ, for ε → 0, as Θ' plus small corrections as we do in the case of the regular limit of Special Relativity Theory, i.e., Classical

Mechanics. The system's behaviour cannot be reduced to the behaviour of the slow sub-system.

What happens at points A and C where the actual phase trajectory leaves the slow manifold DACB and switches to fast motion almost parallel to the x_1-axis? Consider the point A with x_2-coordinate x_2^*. If we take x_2 as the control parameter, we can see that A is a bifurcation point: for $x_2 < x_2^*$ there are *two* equilibrium points for the system (i.e., points at which $\varepsilon x_2' = 0$; cf. equation (6b)), a stable one on DA and an unstable one on AC; at $x_2 = x_2^*$ (point A) the two equilibria merge into one, and for $x_2 > x_2^*$ no equilibria exist. This change in the number of equilibrium points when the system passes through A constitutes a switch to a qualitatively different evolution: lacking a stable equilibrium around A, the system 'jumps' to the available stable equilibrium point for $x_2 = x_2^*$ at B. The bifurcation point corresponds to the point where the system switches from slow to fast motion.

One could object at this point that in a singular perturbation problem, like the van der Pol oscillator in both damping regimes, we just have to reject the decomposition of the full system according to the regular perturbation expansion of x(t); that is, that the system has not been decomposed correctly and that an improved analysis will identify a main sub-system which will be different from the undamped harmonic oscillator or the 'purely slow' sub-system (7a,b) and which will satisfy the uniform limit condition. To what extent and at what costs this can be done, I shall discuss in the next section.

4. Levels of description

In order to find the structure underlying the 'full theory' in a singular perturbation problem, e.g., the van der Pol equation in a certain regime of damping, we can use refined perturbative approaches which allow us to trace the emergence of those features in the higher level ('full') descriptions which are novel with respect to the features of the lower level ('reduced') descriptions.

Due to the mathematical complexity of the van der Pol case, it is easier to see the main point in an example involving an algebraic equation rather than the differential equations we have been discussing. Consider the quadratic equation (Hinch 1991, 4f.):

$$\varepsilon x^2 + x - 1 = 0, \tag{8}$$

with two distinct solutions or roots, $x^{(1)}$ and $x^{(2)}$, which we can write in expanded form as

$$x^{(1)} = (1/2\varepsilon)[- 1 + (1+4\varepsilon)^{1/2}] = 1 - \varepsilon + 2\varepsilon^2 - \ldots ,$$

and

$$x^{(2)} = (1/2\varepsilon)[- 1 - (1+4\varepsilon)^{1/2}] = -1/\varepsilon -1 + \varepsilon - 2\varepsilon^2 + \ldots$$

Suppose we don't have these solutions and we therefore treat (8) for small ε as a perturbation problem. Our expectation is to find the approximate solutions of (8) as corrections of the solution x_0 of the 'unperturbed' (or reduced) problem, i.e., $x - 1 = 0$:

$$x = x_0 + \varepsilon x_1 + \varepsilon^2 x_2 + \ldots \tag{9}$$

The reduced equation, however, has only *one* solution, $x_0 = 1$, and hence the expansion (9) will approximate only $x^{(1)}$. Thus, we have a singular perturbation problem because $\lim_{\varepsilon\to 0} x^{(2)} \neq x_0$. Taking the limit to the reduced problem causes the loss of one solution, $x^{(2)}$, which, for $\varepsilon = 0$, "evaporates off " to $x = \infty$. (This loss of a solution is similar to what happens when the order of a differential equation drops in going to ε=0, like in the case of the heavily damped van der Pol oscillator, Equations (6) and (7) above.)

What happens is that only for $x^{(1)}$ is the εx^2 -term in (8) of 'small' influence and we are justified in neglecting it. The other solution, $x^{(2)}$, is of a different order of magnitude such that for it the εx^2 -term cannot be ignored. To recover this second root, therefore, we introduce a new variable, or rather, a different 'scale' for the variable x; we transform

$$x = x^*/ \varepsilon,$$

which means that, as $\varepsilon \to 0$, the new variable 'shrinks' in comparison with the old variable: $x^*/x \to 0$. This introduction of a new, larger scale for x allows us, so to speak, to catch the second solution before it evaporates off to infinity. For x^*, the original equation (8) becomes

$$x^{*2} + x^* - \varepsilon = 0,$$

which can now be treated as a regular perturbation problem for small ε. The solution is $x^* = -1 - \varepsilon + \ldots$, or, in the old variable, $x = -1/\varepsilon - 1 + \ldots$. We have recovered the lost root $x^{(2)}$.

What is novel at the level of the full equation (8), compared to the features of the reduced equation (9), is the existence of the second solution. It is 'invisible' if we approach the full equation in terms of the (small) scale characteristic of the reduced equation. A new (large) scale is necessary to 'see' the novel feature. No single scale of variable is sufficient to characterize completely the solution of the problem. Both scales, the 'small' x-scale and the 'large' x*-scale, are necessary. This is typical of singular perturbation problems which often are approached with 'multiple scales'. To return to our main example: In the case of the van der Pol oscillator with small damping, we need (at least) two scales for the time-variable, a 'fast' time which characterizes the oscillations (the variable t in the original equations), and a 'slow' time (a variable $T = \eta\, t$) which describes the scale on which the amplitude gradually changes under the influence of the nonlinear damping, that is, the time scale on which the system approaches the limit cycle (Hinch 1991, 116ff.). Two processes are operating simultaneously at different time scales and the regular perturbation approach (which leads to a breakdown of the asymptotic expansion for the amplitude) is not able to identify and discriminate these processes. In the heavy damping regime of the van der Pol oscillator, two different time scales, fast and slow, were also obvious and can be used to generate asymptotic solutions to the dynamical equations (Hinch 1991, 97ff.).

Thus, if we were to reject, as mentioned at the end of the last section, the regular perturbation expansion of the solutions of the van der Pol equation for small damping as leading to a misidentification of the main sub-system and adopt instead a multiple scale expansion, we find a structure underlying the full equation which is different from the ideal harmonic oscillator structure identified through the regular perturbation expansion. But the new structure is characterized by <u>two</u> scales at which the dynamics of the system unfolds while the regular perturbation expansion used the <u>same</u> time scale (t) for all sub-systems $x_{\eta=0}$ (t), ξ_1(t), etc. This means, using the multiple scale approach will not allow us to characterize the higher level features in terms of lower level properties of the system; applying the multiscale expansion, we are unable to remain within the lower level (characterized by time scale t) alone. The singular perturbation approach generates, on its own, different levels of descriptions. Our aim, to correctly identify the main sub-system of the van der Pol oscillator at <u>one</u> level below the full system, has not been achieved. We have to conclude, as before, that the higher level properties are synchronically emergent with respect to the lower level properties.

In the cases considered so far, the scales needed for the asymptotic analysis were generated by the singular perturbation analysis of the differential (or algebraic) equations themselves. If we consider examples of micro-macro-level relations, we encounter again, as mentioned above, singular limits which require asympotic analyses along the lines just discussed. Now, however, the scales of interest are specified by 'the geometry', that is, by the way we distinguish micro and macro according to their typical sizes (cf. Hinch 1991, 126). Assuming that we are dealing with solid macroscopic bodies that have micro structures in the form of periodic arrangements of molecules, the problem can be posed as follows:

> If the period of the structure is small compared to the size of the region in which the system is to be studied, then an asymptotic analysis is called for: to obtain an asymptotic expansion of the solution in terms of a small parameter ε which is the ratio of the period of the structure to a typical length in the region. In other words, to obtain by systematic expansion procedures the passage from a microscopic description to a macroscopic description of the behaviour of the system. (Bensoussan et al. 1978, v)

Introducing the macro scale will reveal the characteristic features of macroscopic bodies, e.g., the 'effective' properties like elasticity of a medium with a specific micro structure like a periodic lattice of atoms.

This is of relevance for a famous example in the philosophical discussions of functionalism, Putnam's case of the round peg of 1 inch diameter which does not fit through a square hole in a board of 1 inch diagonal extension (Putnam 1975, 295ff.; slightly modified). Putnam compares two possible explanations of this fact, a microscopic one in terms of the arrangement of molecules in peg and board, etc. ('the microstructural deduction'), and a macroscopic one in terms of the geometrical relations between the macroscopic objects involved. The interesting observation to be made is that the 'microstructural deduction' is an instance of the kind of asymptotic analysis considered above. We can expect a singular limit relation between the micro and macro descriptions and thus the emergence of novel features at the higher (macro) level which require the introduction of a new scale into the lower level (micro) description.

The nonuniformity of the limit is the real reason for why the macro description tells us 'something different' than the micro description, and for why the former is not reducible to the latter

— the macro description is not a straightforward special case of the more general micro view. One extremely important service that the asymptotic expansion of the micro description in terms of two different scales does for us is to make it clear to what extent the macro description, the behaviour of the system at the macro scale, is independent of, or insensitive to, the details of the system's behaviour at the micro scale. For instance, in calculating the effective elastic moduli of a medium, it may turn out that these quantities, in a first approximation, do not depend on the micro scale variables but only on the macro variables. In the terms that are familiar from the philosophical discussion: the 'multiple realizability' of the macro description, which Putnam mentions as its main advantage over the micro story, is an immediate outcome of the asymptotic procedure described.[10]

Conclusion

I have explicated two notions of emergence which are based on two ways of distinguishing levels of properties for dynamical systems. Once the levels are defined, the strategy of characterizing the relation of higher level to lower level properties as diachronic or synchronic emergence is the same. In the diachronic case we simply compare the behavioural properties of the system at a time (lower level) with those at a later time (higher level). In the synchronic case we decompose the system (or, rather, its behaviour: the higher level) into a combination of lower level sub-systems (or, rather, the behaviour generated by them) which are identified through a perturbation analysis of the full system. In each case, the higher level properties are said to be emergent if they are 'novel' or 'irreducible' with respect to the lower level properties. Novelty and irreducibility are given precise meanings in terms of the effects that the change of a (bifurcation or perturbation) parameter in the system has. (The same strategy can also be applied to other ways of separating levels of properties, like the micro/macro distinction.)

The notions of emergence developed here are notions of emergence in a weak sense: the higher level emergent properties we capture are always structural properties (or are realized in such properties), that is, they are defined in terms of the lower level properties and their relations. Diachronic and synchronic emergent properties are distinctions within the category of structural properties. They do not describe properties with causal powers 'over and above' the causal powers structural properties have in virtue of being configurations of their lower level constituents.[11] Emergentists may find such weak emergence pointless (cf. O'Connor 1994). But weak emergence, I believe, is all we get if we try to explicate a notion of emergence that is neither so strong that it has no application at all, nor so weak that it renders more or less every property emergent.

Appendix I: Essential Interaction

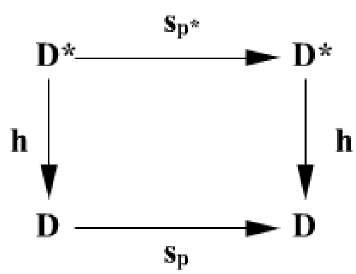
Emergent properties are often supposed to result from 'essential interactions' of constituents of a system, not from the mere fact of having the constituents collected in a heap (cf. Humphreys 1996, or Wimsatt's condition (iv) above). This condition for emergent properties can be given an intuitively appealing interpretation within the framework of dynamical systems theory — at first glance, at least.

A dynamical system with N degrees of freedom (which needs 2N generalized coordinates to describe it) is said to be (completely) integrable if it has N 'first integrals of motion' for all initial conditions and parameter values. Intuitively this means that if a system is integrable, we can always find a representation of it (by transforming the original description of the system) that describes a collection of N non-interacting or uncoupled constituent systems. Thus, we could define an interaction of components of a system as not essential if there is a way of transforming the description of the system into the description of a system of 'free' components which change independently of each other, i.e., without interacting. Essential interaction occurs in a system if the actually interacting components cannot be replaced by an equivalent system of non-interacting parts.

Note that on this suggestion, essential interaction is not as closely connected to nonlinearity of a system as is sometimes thought: there are nonlinear systems which are integrable and hence would lack essential interaction. (Non-integrability of a dynamical system, however, is a necessary, though not sufficient condition for the occurrence of chaotic behaviour in a system.) The problem with this proposal is that the notion of integrability has a several quite distinct meanings which I suppressed in the characterization above. Traditionally, integrability was defined only for Hamiltonian systems, that is, dissipative systems would automatically be excluded from the range of applicability of the concept. Such a restriction would obviously not be useful in capturing the intuitive sense of essential interaction. More recently, integrability has been applied (along the lines of the above definition) to dynamical system in general, dissipative or not; furthermore, degrees of integrability have been distinguished from complete integrability. The coordinate transformations that are admissibile under such wider notions of integrability include 'nonlocal' transformations which are, for our purposes, somewhat opaque because it is not clear in what sense a nonlocal coordinate transformation does not implicitly involve interaction between components. The question, therefore, is whether there are technical senses of integrability which are neither too restrictive nor too wide in order to capture the intuitive notion of essential interaction.[12]

Appendix II: Structural Stability

We say that a dynamical system Σ, considered as the transformations s_p: D → D on the system's phase space of initial conditions x at some initial time into solutions $s_p(x)$ of the dynamical equations at other times, is topologically equivalent to another system Σ* with s_{p*} : D* → D* if there exists a homeomorphism h (a one-to-one mapping continuous in both directions) of the phase space trajectories of the first system onto the trajectories of the second such that the diagram



commutes, that is:

$$h\,[s_p(x)] = s_{p*}\,[h(x)].$$

In other words: two systems are equivalent in this sense if the change from s_p to s_{p*}, introduced by the variation of the control parameter p, can be compensated by a transformation (h) of the coordinates. A system Σ is structurally stable if every system 'close' to Σ is topologically equivalent to Σ. (The notion of closeness has to be spelled out in whatever topology is imposed on the phase spaces. Usually one postulates that a map *f* is close to a map *g* if *g* belongs to an ε-neighbourhood of *f* such that every map in that neighbourhood agrees with *f* and its derivatives up to ε > 0.)[13]

In the van der Pol case, the corresponding diagram would not commute if Σ were the system without damping ($\eta = 0$) because in any neighbourhood of Σ (η slightly different from 0) there will be systems which are topologically inequivalent to Σ because they have limit cycle behaviour, a pattern of trajectories which cannot be generated from the phase space portrait of Σ by a mere transformation of coordinates.[14]

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Endnotes

1. But there is controversy about this: see O'Connor 1994; McLaughlin 1992; Stephan 1992.

2. On how to think of properties in physics, cf. Wilson 1993, 75ff., and Wilson 1985 for further critical discussion of functional properties.

3. For an overview of the applications of van der Pol's equation as well as for methods of solving it, see Grasman 1987, ch.1.

4. For an informal introduction to structural stability, see, e.g., Saunders 1980, 17-21. A somewhat more technical description is given in Appendix II.

5. For a proposal of how failure of structural stability is connected to failure of explainability,

that is, for how the unexplainability requirement for emergent properties can be reconstructed in the present framework, see Rueger (ms.).

6. Cf. Liu 1998; for the singular limits of wave optics and of Quantum Mechanics, see Batterman 1995.

7. Even though, for any fixed t, the expansion converges. Cf. Hinch 1991, 116.

8. In a recent paper on the bifurcation behaviour of equations modelling lasers, for instance, the authors perform a (singular) perturbation analysis of the full set of equations, labelling the ($\varepsilon = 0$)-equations as describing “a fast subsystem” of the whole system (Khibink et al. (1998), 298).

9. This is similar to the notion of ‘structure-restricted realization’ in Kim 1992, 5-8.

10. More on this in Batterman (forthcoming).

11. Cf. similar remarks in Bedau 1997, 394f., although Bedau’s notion of weak emergence is quite different from mine.

12. For an overview of the various senses and applications of integrability, see Tabor 1989, especially ch.8.

13. Adapted from Arnold 1983, ch.3; cf. also Guckenheimer/Holmes 1983, 38f.

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